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A TENTATIVE THEORY OF METALLIC WHISKER GROWTH

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Peach's[1] very pretty explanation of the formation of metalic whiskers[2] seems to be ruled out by the observation[3] that they grow at the root. The growth seems to be influenced[4] by the atmosphere over the surface. The energy required to form a fresh surface may be more than repaid if it is attacked avidly enough by the atmosphere; there is then an effective negative surface tension $-\gamma$. We might have perhaps $\gamma b^2 \sim 1\,ev$ (b = lattice constant). The ratio γ/μ (μ = shear modulus) would then be about 1A. The surface tension forces on a small hump on the surface (fig. 1 (a)) obviously have the right character (a central pull surrounded by a restraining pressure) to 'wire-draw' it into a whisker according to intuitive ideas of plastic flow. For the observed whisker size ($R \sim 10^{-4}$ cm) the stress of order γ/R in and just below the hump might exceed the actual yield stress, though not the theoretical yield stress ($\sim \mu/20$). However, on this small scale one must consider in detail how flow is catalyzed by dislocation.

The model of fig. 1 lends itself to rough calculations. A Frank-Read source of length l and vertical Burgers vector b lies in a horizontal plane (b) at a depth of order l below the hump. The stress τ_{zz} (fig. 1(c)) makes the source emit a dislocation loop by 'climb'. The loop expands in the plane (b) until it reaches a radius where $\tau_{zz} = 0$. Here the stress τ_{rz} assisted by image forces makes the loop glide vertically, so adding one atomic layer to the base of the hump. When by repetition of this a reasonable whisker has grown τ_{zz} will be $2\gamma/R$ in the whisker and about γ/R at the source. To operate the source

τ_{zz} must be at least $\mu b/l$. With $\gamma/\mu \sim 1A \sim b$ this will be so if $R \sim l$. The stress at the source ultimately falls off both for $R \gg l$ and $R \ll l$ if the source depth stays constant. The whisker radius is thus tied to the length of a Frank-Read source, which is usually supposed to be about one micron. The surface $\tau_{zz} = 0$ forms a 'stress funnel' which guides each loop more or less unerringly to break surface at the base of the whisker, and so keeps its diameter constant. If the motion of the source is not to be stopped by the back-pressure of the vacancies it emits there must be suitable sinks for them. It can be shown that surface tension changes the volume of a body of any shape with compressibility χ by $\frac{2}{3}\gamma\chi$ times its surface area. For a macroscopic specimen the corresponding mean pressure $\frac{2}{3}\gamma \times$ (surface/volume) would fall far short of the value required to make Frank-Read sources in the interior act as the necessary sinks. However, it should not be hard to find them at the surface. Frank[5] has shown that even with positive surface tension it may be energetically an advantage for a dislocation reaching the surface to develop a hollow core. Or again, if we had a depression instead of a hump in fig. 1 the source would work in reverse, absorbing vacancies and deepening the depression.

We may use a calculation of Mott's[6] to find the rate of growth. He showed that, if a cube has normal stresses P on one pair of opposite faces and -P on another pair, a volume $V \sim NbD(Pb^3/kT)$ of material is transferred from one pair of faces to the other in unit time if there are N points on dislocations which can absorb or emit vacancies and the coefficient of self-diffusion is D. Our case is analogous, P is γ/R times a factor K depending on the

detailed stress-distribution, including a possible stress concentration if successive loops help one another N is about l/b, the number of lattice sites per loop times the fraction β (perhaps $\ll 1$) of them which can emit or absorb vacancies times n the number of loops in transit between source and surface at one time. The rate of change of the whisker length h will thus be

$$\dot{h} = V/\pi R^2 \sim K \beta n D \, (b/l^2)(\gamma b^2/kT).$$

With the value of D for tin at room temperature[7] we can get a growth rate of a millimeter or a centimeter per year with $K\beta n \sim 100$ or 1000. The small number of accidental coincidences of sources and suitable surface irregularities may be enough to account for the number of whiskers per unit area. If not, we might suppose that the sources build their own humps by operating initially without stress as the result of a subsaturation of vacancies due to a change of temperature.

Many variations of this model are possible. The transfer of loops to the surface might occur by the formation and joining up of secondary loops in a vertical plane as in 'prisimatic punching'[8], where the stress distribution is similar. The whiskers might then be prisimatic. Professor Seitz (to whom the writer is indebted for helpful discussions) has suggested a mechanism involving a spiral prisimatic dislocation[9] which is the internal counterpart of spiral growth on the surface and which leaves a screw dislocation along the axis of the whisker.

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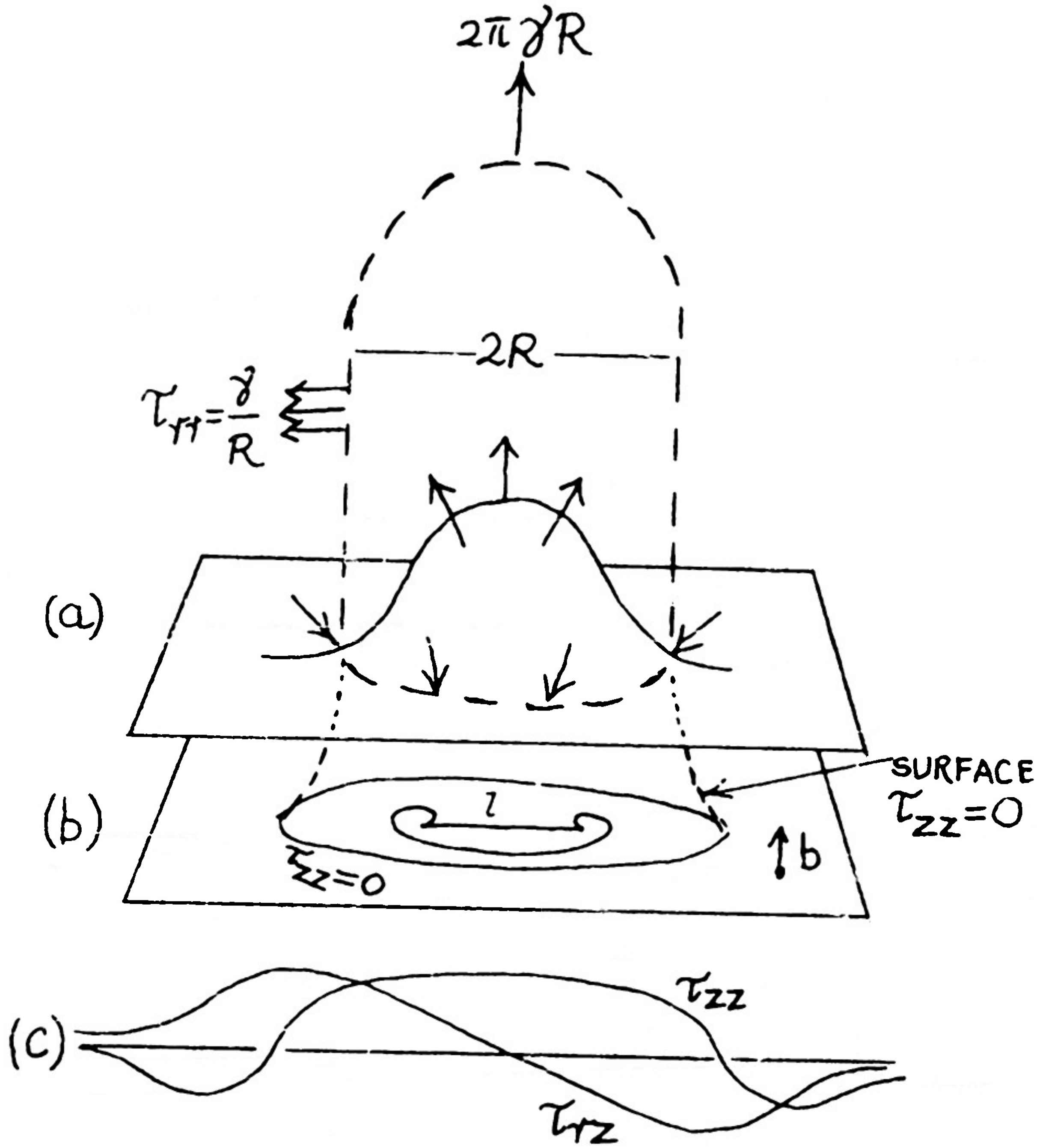

$2\pi\gamma R$
$2R$
$\tau_{rr}=\frac{\gamma}{R}$
(a)
(b)
l
$\tau_{zz}=0$
b
SURFACE
$\tau_{zz}=0$
(c)
τ_{zz}
τ_{rz}

FIG. 1